CRITICAL RESISTANCE AND CRITICAL FLUIDIZATION VELOCITY OF FINE-GRAINED MATERIAL IN CONICAL EQUIPMENT

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The problem of the determination of the critical resistance and critical fluidization velocity of fine-grained material in conical equipment is investigated analytically. The relations obtained are compared with experiment.

Existing formulas [1, 2, 4, 5, 6] for the minimum fluidization velocity of fine-grained material in conical equipment and the maximum resistance of the bed are based on experimental data and lack the generality inherent in analytical solutions. Analytical formulas giving satisfactory agreement with the experimental data are not known to the authors.

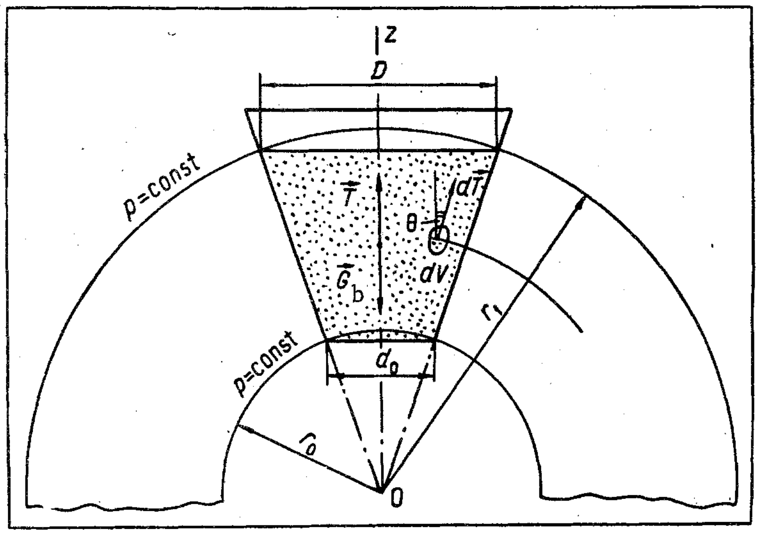

Fig. 1. Geometrical representation of the pressure field in conical equipment.

Let us examine the passage of a gas through fine-grained material in a conical apparatus. Replacing the plane surfaces by spherical ones (Fig. 1), we get a sector of a sphere filled with material. We shall consider this to be part of the spherical layer lying between two concentric spheres of radii r_1 and r_2. If a fluid issuing from a point source at the center 0 propagates through this layer, the mass flow of fluid over any spherical surface will be the same. We shall assume that the fluidizing agent is a viscous incompressible fluid, that the process is steady, and that the flow is potential. Then the equipotential surfaces of the pressure field in the flow (isobars) coincide with the spherical surfaces. Consequently, the pressure and velocity in the volume filled with flowing fluid depend only on radius r. This applies in full measure to the spherical sector under examination when the influence of the cone walls on the fluid flow can be neglected.

According to Ergun's formula [7],

$$dp = \frac{2\rho w^2 (1-\varepsilon)}{\varepsilon^3 d}\left[75\frac{(1-\varepsilon)\nu}{wd} + 0.875\right]dr. \tag{1}$$

Here w is the velocity of the fluidizing agent at an arbitrary r section of the layer, calculated for the total cross section of the apparatus (without the material). The assumption of incompressibility of the fluidizing agent (ρ = const, $\operatorname{div} w = 0$) allows us to relate this value to the velocity w_0 at the bottom section of the apparatus by the obvious relation

$$w/w_0 = r_0^2/r^2. \tag{2}$$

Substituting the expression for w from (2) into (1), we obtain a differential equation for the pressure field.

Because of the appreciable adhesion between particles, fluidization in conical equipment, as in cylindrical, begins when the bed resistance equals the weight of material in it. The layer of material is suspended as a whole by the action of the flow, and only after the adhesion between particles is overcome by fluidization does the flow force a channel for itself in the center of the equipment, leading to a corresponding redistribution of the velocity field. The action of the fluid on the bed of material is determined by the pressure forces applied to the surfaces bounding the material. Therefore,

$$\mathbf{T} = -\oint_s p\,d\mathbf{s} = -\int_V \operatorname{grad} p\,dv = +\mathbf{G}_b. \tag{3}$$

Due to the symmetry of the pressure field about the cone axis, the total force T acting on the layer due to the fluid flow [6] is directed vertically upwards. The contribution of forces acting on volume elements dv to the total force T is proportional to the cosine of the angle between the z axis and the direction of the force, which coincides with the direction of $\vec{\nabla} p$, i.e., with the direction of the radius vector at the point in question.

Taking this into account and putting $|\operatorname{grad} p| = dp/dr$, we have

$$T = -\int_V \frac{dp}{dr}\cos\vartheta\,dv = \pi\sin^2\frac{\alpha}{2}(r_1 - r_0)\left[Aw_0 + \frac{Bw_0^2}{r_1 r_0}\right], \tag{4}$$

where

$$A = \frac{150\,\rho\,r_0^2(1-\varepsilon)^2\nu}{\varepsilon^3 d^2}; \quad B = \frac{1.75\,\rho\,r_0^4(1-\varepsilon)}{\varepsilon^3 d}.$$

The weight of the bed is determined by the product of its volume and its bulk weight (allowing for the action of the expulsive force):

$$G_b = Vg(\rho_M - \rho)(1-\varepsilon) = \frac{4}{3}\pi g(\rho_M - \rho)(1-\varepsilon)(r_1^3 - r_0^3)\sin^2(\alpha/4). \tag{5}$$

Equating (4) and (5) and solving the resulting quadratic equation with respect to w_0, we find

$$w_0 = \frac{150(1-\varepsilon)\nu r^*}{3.5d}\left[\left(1 + \frac{7\mathrm{Ar}\,\varepsilon^3(r^{*2} + r^* + 1)}{150^2(1-\varepsilon)^2 3r^*\cos^2(\alpha/4)}\right)^{1/2} - 1\right], \tag{6}$$

where

$$r^* = r_1/r_0 = D/d_0.$$

Transforming (5), we obtain a formula for determining Re_0^{co} in conical equipment:

$$\mathrm{Re}_0^{co} = 2\mathrm{Ar}\left\{\frac{150(1-\varepsilon)}{\varepsilon^3}M\left[1 + \left(1 + \frac{7\mathrm{Ar}\,\varepsilon^3}{150^2(1-\varepsilon)^2 r^* M}\right)^{1/2}\right]\right\}^{-1} \approx$$
$$\approx \mathrm{Ar}\left[\frac{150(1-\varepsilon)}{\varepsilon^3}M + \left(\frac{1.75}{\varepsilon^3}\frac{M}{r^*}\right)^{1/2}\sqrt{\mathrm{Ar}}\right]^{-1}, \tag{7}$$

where

$$M = 3\cos^2(\alpha/4)/(r^{*2} + r^* + 1).$$

For cylindrical chambers Re_0^{cy} is determined from the formula given in [3]:

$$\mathrm{Re}_0^{cy} = 2\mathrm{Ar}\left\{\frac{150(1-\varepsilon)}{\varepsilon^3}\left[1 + \left(1 + \frac{7\mathrm{Ar}\,\varepsilon^3}{150^2(1-\varepsilon)^2}\right)^{1/2}\right]\right\}^{-1} \approx$$
$$\approx \mathrm{Ar}\left[\frac{150(1-\varepsilon)}{\varepsilon^3} + \left(\frac{1.75}{\varepsilon^3}\right)^{1/2}\sqrt{\mathrm{Ar}}\right]^{-1}. \tag{8}$$

The ratio of critical velocities (limit of stability) in conical and cylindrical equipment for the case of small Ar ($\mathrm{Ar} < 10^3$) is

$$\frac{w_0}{w_0^{cy}} \approx \frac{1}{M} = \frac{r^{*2} + r^* + 1}{3\cos^2(\alpha/4)}, \tag{9}$$

and for large Ar ($\mathrm{Ar} > 10^6$)

$$\frac{w_0^{co}}{w_0^{cy}} \approx \left(\frac{r^*}{M}\right)^{1/2} = \left(\frac{r^*(r^{*2}+r^*+1)}{3\cos^2(\alpha/4)}\right)^{1/2}. \qquad (9')$$

The errors in using the approximate formulas (9) and (9') do not exceed 10% for $r < 10$ in the corresponding ranges of variation of Ar.

The porosity of the undisturbed bed was taken to be 0.4 on the average. It is known that at the moment of fluidization the bed volume increases by ≈10%. Assuming the inflation of the bed to be uniform, we take $\varepsilon = 0.45$ and have

$$\frac{w_0^{co}}{w_0^{cy}} \approx \frac{1+(1+10^{-4}\,\mathrm{Ar})^{1/2}}{1+(1+10^{-4}\,\mathrm{Ar}/Mr^*)^{1/2}} \; \frac{1}{M}. \qquad (10)$$

Figure 2 shows calculated values of w_0^{co}/w_0^{cy} compared with experimental data taken from [1]; there is agreement within the limits of experimental error. The more notable divergence at $r = 6$ may be explained by the appearance of a gas channel in the narrow section of the cone.

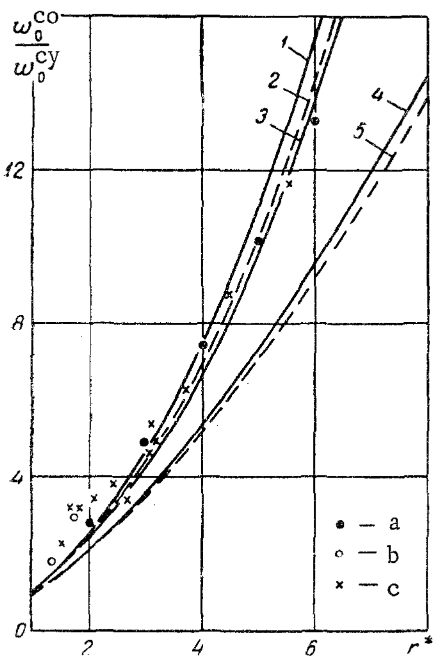

Fig. 2. Comparison of calculated values of w_0^{co}/w_0^{cy} with experimental data of [1], and limits of variation of w_0^{co}/w_0^{cy} in various flow regimes: 1) Ar → 0; α = 60°; 2) 0, 10°; 3) 10^3, 60°; 4) ∞, 60°; 5) ∞, 10°; a, b, c) experimental points from [1].

An empirical relation was obtained in [6] for the conditional value of the critical spouting velocity, calculated for the bottom stationary cross section of the equipment:

$$\mathrm{Re} = 0.174\,\mathrm{Ar}^{0.5}\left(\mathrm{tg}\,\frac{\alpha}{2}\right)^{-1.25}\left(\frac{D}{d_0}\right)^{0.85}. \qquad (10')$$

It should be borne in mind that the velocity w_s determined from (10') corresponds to the moment of formation of an external spout, while Eq. (7) of this paper is based on the velocity w_0^c corresponding to peak pressure. When $\mathrm{Ar} = 10^4$, $\alpha = 40°$, the ratio w_s/w_0^{co} varies from 3.7 to 1.55 with variation of $r^* = D/d_0$ from 2 to 7.

A formula for the bed resistance at the moment of commencement of fluidization may be obtained from (3) with $r = r_0$, since

$$\Delta p = p|_{r=r_0} - p|_{r=r_1} = p|_{r=r_0},$$

$$\Delta p = \frac{\rho w_0 r_0 (1-\varepsilon)}{\varepsilon^3 d} \times$$

$$\times\left[\frac{150(1-\varepsilon)\nu}{d}\,\frac{r_1-r_0}{r_1} + \right.$$

$$\left. + \frac{1.75\,w_0(r_1^3-r_0^3)}{3r_1^3}\right],$$

or in dimensionless form

$$\Delta p^* = \frac{\Delta p}{g(\rho_M-\rho)(1-\varepsilon)(r_1-r_0)} = \frac{\Delta p}{\gamma_n h_0} =$$

$$= \frac{150(1-\varepsilon)}{\varepsilon^3 r^*}\,\frac{\mathrm{Re}_0^{co}}{\mathrm{Ar}}\left[1+\frac{1.75}{450}\,\frac{r^{*2}+r^*+1}{r^{*2}(1-\varepsilon)}\,\mathrm{Re}_0^{co}\right]. \qquad (11)$$

Here Re_0^{co} is determined from (7).

When Ar is small ($<10^3$), Δp^* may be calculated from the simpler formula obtained from (11):

$$\Delta p^* \approx \frac{r^{*2}+r^*+1}{3r^*\cos^2(\alpha/4)}. \qquad (12)$$

When $\mathrm{Ar} > 10^7$

$$\Delta p^* \approx \left[\frac{r^{*2}+r^*+1}{3r^*\cos^2(\alpha/4)}\right]^2. \qquad (13)$$

The errors in calculating Δ_p^* from the approximate formulas (12) and (13) in the corresponding ranges of variation of Ar do not exceed 10% for $r < 10$ in comparison with calculations from (11).

In Fig. 3 the theoretical relation $\Delta p^* = \Delta p^*(r^*)$ is compared with experimental data from [1] and [5]; also shown are the limits of variation of Δp^* in various flow regimes. Some disagreement between theory and experiment is evidently attributable to the influence of the cone walls on the flow and to the deviation of the actual pressure field from the idealized picture assumed.

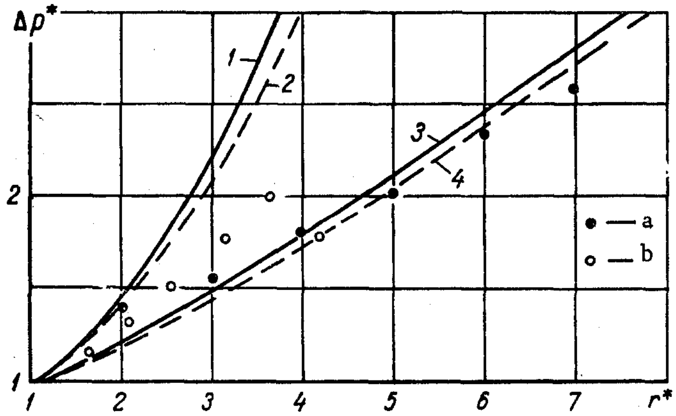

Fig. 3. Comparison of values of peak pressure Δp^* calculated from formulas (11) and (14) and limits of variation of $\Delta p^*(r^*)$ in various flow regimes: 1, 2) $Ar \to \infty$, $\alpha = 60°$ and 10°; 3, 4) $Ar \to 0$, $\alpha = 60°$ and 10°; a) from (14); b) from [5].

The experimental data from [1] correspond to $Ar \simeq 10^3$, and are satisfactorily approximated by the theoretical curve drawn from Eq. (11) of this paper (the maximum deviation does not exceed 15%). A comparison has also been made with the empirical relation obtained in [1]*:

$$\Delta p^* = 1 + \frac{\Delta\pi}{\gamma_n h_0} = 1 + 0{,}062 \left(\frac{D}{d_0}\right)^{2{,}54} \times \left(\operatorname{tg}\frac{\alpha}{2}\right)^{-0{,}18} \left(\frac{D}{d_0} - 1\right)^{-1}. \quad (14)$$

Lack of complete data on the material used in the experiments of [5] prevented us from making a similar comparison with theory. The experimental points derived from the data of [5] fall within the limits of values of Δp^* calculated theoretically.

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* In formula (14) of [1] the exponent −1 of $D/d_0 - 1$ was omitted. The curve of Fig. 4 from [1] corresponds to relation (14) with the exponent −1.